Turn in the following problems:

1. The point $P(0.5, 0)$ lies on the curve $y = \cos(\pi x)$.

   (a) If Q is the point $(x, \cos(\pi x))$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x:

      i. 0
      ii. 0.4
      iii. 0.49
      iv. 0.499
      v. 1
      vi. 0.6
      vii. 0.51
      viii. 0.501

   (b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(0.5, 0)$.

   (c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(0.5, 0)$.

   (d) Sketch the curve, two of the secant lines, and the tangent line.

2. Fill in the blank with "all", "no", or "some" to make the following statements true.

   - If your answer is "all", explain why.
   - If your answer is "no", give an example and explain.
   - If your answer is "some", give two examples that demonstrate when the statement is true and when it is false. Explain your examples.

   Note: An example must include either a graph or a specific function.

   (a) For _______ functions f and g, if $\lim_{x\to a} g(x) = 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist.

   (b) For _______ functions g, if $\lim_{x\to a^+} g(x) = 2$ and $\lim_{x\to a^-} g(x) = -2$, then $\lim_{x\to a}(g(x))^2$ exists.

   (c) For _______ functions f and g, if $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a}(f(x) + g(x))$ exists, then $\lim_{x\to a} g(x)$ exists.

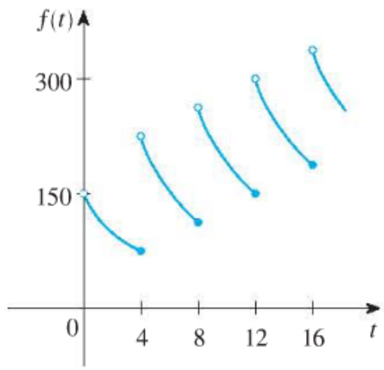
   (d) For _______ real values of x, the functions $f(x) = \frac{x^2 - 9}{x - 3}$ and $g(x) = x + 3$ are equal.

   (e) For _______ real numbers a, $\lim_{x\to a} f(x) = f(a)$

3. A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find

$$\lim_{t\to 12^-} f(t) \text{ and } \lim_{t\to 12^+} f(t)$$

and explain the significance of these one-sided limits.



4. Sketch the graph of an example of a function f that satisfies all of the given conditions.

- $\lim_{x\to 1} f(x) = 2$
- $\lim_{x\to 3^-} f(x) = -4$
- $\lim_{x\to 3^+} f(x) = 4$
- $f(1) = 0$
- $f(3) = 4$

5. Sketch the graph of the function and use it to determine the value of a for which $\lim_{x\to a} f(x)$ does not exist.

$$f(x) = \begin{cases} 1 + \sin(x) & \text{if } x < 0 \\ \cos(x) & \text{if } 0 \le x \le \pi \\ \sin(x) & \text{if } x > \pi \end{cases}$$

6. (a) What is wrong with the following equation?

$$\frac{x^4 - 8x^2 + 16}{x^2 - 4} = x^2 - 4$$

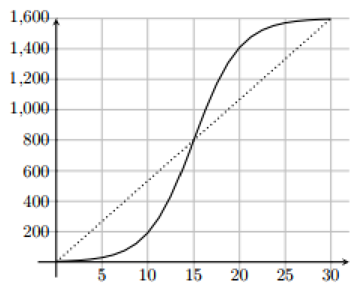
(b) In view of part (a), explain why the equation

$$\lim_{x \to 2} \frac{x^4 - 8x^2 + 16}{x^2 - 4} = \lim_{x \to 2} (x^2 - 4)$$

is correct.

These problems will not be collected, but you might need the solutions during the semester:

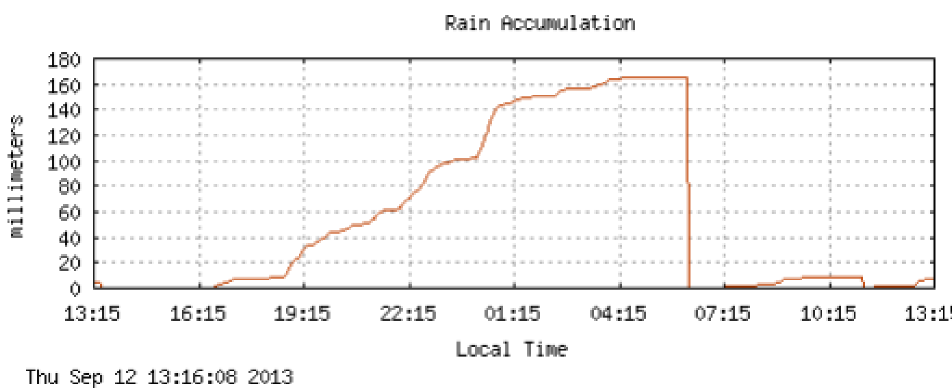
1. The solid curve in the graph below gives position s of a car along a straight roadway (measured in meters), as a function of time t (measured in seconds).



   (a) Find the slope of the dotted line in the graph above. Explain (including units), what this slope represents.

   (b) Estimate the instantaneous velocity at $t = 15$. Include units. Draw and label the line you used to estimate this.

2. Below is a plot of the rainfall accumulation from the 2013 Boulder flood taken from the Foothills Lab Weather Station. The rainfall is measured in millimeters.



   (a) Use the graph to estimate the average rainfall rate between 4:15 pm (marked as 16:15 on the graph) and 4:15 am the next morning (marked as 04:15 on the graph). Show all work and include units. Draw the line that you are finding the slope of.

   (b) When is it raining hardest? Explain how you know.

   (c) Estimate the rainfall rate at 22:15 (include units). Draw the line that you are finding the slope of.

   (d) What does the graph indicate is happening to the rainfall during the hour after 4:15 am?

   (e) Explain the precipitous drop between 04:15 and 07:15.

Optional Challenge Problem

Complete 5g from Project 1